

Tailoring a Global Harmonic Model Xgm2016 to the Territory of Egypt

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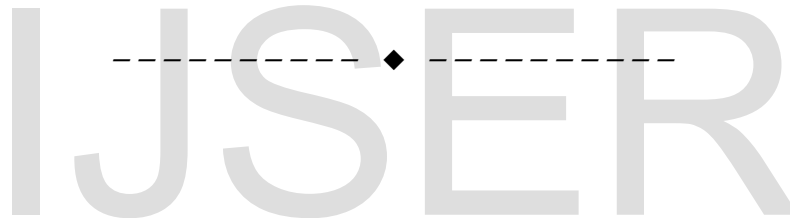
Abstract — The aim of this research is to tailor the Experimental Gravity Field Model [XGM2016] harmonic model to Egypt for better modeling of the Egyptian gravity field. This can be made by computing the differences between local gravity anomalies and those derived from the geopotential model, then the harmonic analysis of the residual gravity anomalies yields correction terms that are added to the original spherical coefficients of the relevant model to give the final modified coefficients of the fitted model.

Several methods can be used to achieve the tailoring process, in this paper; we have used the integral formulas, suggested by (Weber and Zomorrodian, 1988).

In this study XGM2016 has been tailored to fit the gravity data in Egypt using integral formulas in order to be used for the reference gravity field model for the Egyptian territory, according to its superior performance in previous study (Moamen et. al, 2017).

The results illustrated that the tailored model of XGM2016 denoted as EGXGM2016 has perfect performance, where its mean value is [1.56 mgal], the standard deviation is [10.50 mgal] and the range of the reduced gravity anomalies to EXGM2016 compared with XGM2016 have lesser values by about [43.52 mgal] and the stander deviation is better by 62%. While the external accuracy by using [71 free air gravity anomaly data points] denoted that 54% lesser in terms of stander deviation.

Keywords— EGMs, Integral technique Geopotential models, gravity, gravity anomalies, enhancement EGMs, Tailoring EGMs.



1. INTRODUCTION

By using a global harmonic model in local geoid solutions, the respective low frequency features would only be reliable, if the model contains local gravity data from the region under consideration (Shaker et. al, 1997). Concerning Egypt, the Model [XGM2016] harmonic model, as all other models, is claimed to lack in the Egyptian terrestrial gravity data (Amin, 2002). Therefore, the long wavelength features for the Egyptian territory cannot be optimally recovered from such global models, thus degrading the target precision of the local geoid solution.

Therefore, Global Geopotential Models (GGMs) are not perfect due to imperfect distribution, density, and accuracy of the available global heterogeneous gravity data, whereas data availability and data accuracy can only be enhanced by performing additional observations, accordingly the resolution of the geopotential models can then be improved by increasing its maximum degree.

The Egyptian mean free-air gravity anomalies are used to estimate the harmonic coefficients of the tailored model XGM2016 denoted as EXGM2016 complete to degree and order 469.

2. BACKGROUND AND METHODOLOGY

Tailoring a specific harmonic model to the local data in a certain region utilizes a local harmonic analysis scheme, which uses the respective data window as input. This procedure amounts to using the original model in a remove-restore procedure and predict an equivalent set of harmonic coefficients corrections up to the model's maximal degree 469 and coefficients of higher degrees up to the maximum possible resolution (Wenzel, 1998). In particular, if the input data are free air gravity anomalies, Δg , then the XGM2016 low frequency geoid part is removed to obtain the residual data δg ,

Improving or refining a geopotential model to fit the gravity field of the certain region using additional gravity data relevant to that area, are often referred to as tailoring the same model to this region. The basic assumption is that the additional gravity data have not been used originally in the development of the geopotential model. To achieve this process, the differences between the additional gravity data

and those obtained from the geopotential model of interest a region, EGYPT, are used in harmonic analysis techniques to obtain correction terms that are added to the coefficients of the original model to give the final refined coefficients of the tailored model as follows:

The gravity anomaly (Δg) in spherical approximation is given (Torge, 1989, p. 44) as follows:

$$\Delta g(r, \theta, \lambda) = \frac{GM}{r^2} \left[\sum_{n=2}^{n_{max}} (n-1) \left[\frac{a}{r} \right]^n \sum_{m=0}^n \bar{C}_{nm}^* \cos m\lambda + S_{nm} \sin m\lambda P_{nm} \sin \varphi \right] \quad (1)$$

Where:

GM is the geocentric gravitational constant;

n_{max} is the maximum degree;

n, m is the degree and order;

\bar{C}_{nm}^* is the relevant fully normalized spherical harmonic C-coefficients of degree n and order m , reduced for the even zonal harmonics of WGS-84 reference ellipsoid

\bar{S}_{nm} is the relevant fully normalized spherical harmonic S-coefficients of degree n and order m ,

$\bar{P}_{nm}(\sin \varphi)$ is the fully normalized associated Legendre function of degree n and order m ,

φ, λ the geocentric latitude and longitude;

γ the normal gravity;

a the scaling factor and r is the geocentric distance.

Ψ the geocentric latitude,

λ the geodetic longitude,

r the geocentric radius,

The quadrature procedure for estimating spherical harmonic coefficients may be computed from gravity anomalies Eq. (1)

$$\left\{ \begin{array}{l} \bar{C}_{nm} \\ \bar{S}_{nm} \end{array} \right\} = \frac{1}{4\pi} \iint_{\sigma} \frac{r^2}{GM} \left(\frac{r}{a} \right)^n \frac{1}{n-1} \Delta g(r, \theta, \lambda) \left\{ \begin{array}{l} \cos m\lambda \\ \sin m\lambda \end{array} \right\} \bar{P}_{nm}(\cos \theta) d\sigma \quad (2)$$

Where σ is a unit sphere and $d\sigma$ is a surface area element.

The actual method of evaluation of Eq. (2) is carried out using a set of mean gravity anomalies all over the earth's surface, however, when the integration is carried out over a local area only, it is thus implicitly assumed that the mean gravity anomalies are equal to zero outside σ .

A mean gravity anomaly can be computed from geopotential models, (Rapp, 1977, p.4)

as follows:

$$\overline{\Delta g}_{Model} = \frac{GM}{r^2} (n-1) \left[\frac{a}{r} \right]^n \beta_n \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + S_{nm} \sin m\lambda P_{nm} \cos \theta) \quad (3)$$

Where β_n are the Pellinen smoothing functions that can be evaluated from a recurrence relation formula derived in (Sjöberg, 1980).

The obtained $\delta \overline{\Delta g}$ values from [equation 4] are then used as input for the harmonic analysis algorithm [equation 5] to receive the harmonic coefficients corrections $\delta \overline{C}_{nm}$ & $\delta \overline{S}_{nm}$. The coefficients' corrections up to degree and order 469 are then restored back to the XGM2016 Eq. (6) relevant coefficients, in order to end up with the relevant tailored coefficients,

$$\delta \overline{\Delta g} = \overline{\Delta g}_{Terrestrial} - \overline{\Delta g}_{Model} \quad (4)$$

$$\begin{Bmatrix} \delta \overline{C}_{nm} \\ \delta \overline{S}_{nm} \end{Bmatrix} = \frac{1}{4\pi} \iint_{\sigma_{GM}} \frac{r_i^2}{(a)^n} \frac{1}{n-1} \frac{1}{\beta_n} \delta(\overline{\Delta g}_i) \iint_{\Delta\sigma_i} \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \overline{P}_{nm}(\cos\theta) d\sigma \quad (5)$$

Finally, the coefficients of the tailored model are now obtained by adding the corrections Eq. (5) to original coefficients of Eq. (2), then the residual gravity anomalies Eq. (5) may once again be formed iteratively (Kearsley & Forsberg, 1990) as follows:

$$\begin{Bmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{Bmatrix}_{EGEXGM2016} = \begin{Bmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{Bmatrix}_{XGM2016} + \begin{Bmatrix} \delta \overline{C}_{nm} \\ \delta \overline{S}_{nm} \end{Bmatrix}_{Corrections} \quad (6)$$

Logically, the resulting terms with $n > 719$ represent the coefficients themselves. Of course, the actual spectral content inherent into the data judges the maximum degree and order of the significant and reliable terms that could be extracted. The maximum degree and order depends also on the used technique for extracting them.

3. THE AVAILABLE DATA

The gravity anomaly data, Fig. [1] shows an irregular distribution with large gaps, especially on land while the coverage of the Mediterranean and Red Sea is rather good than the land covering.

The local gravity data used in this study were grouped into two sets as shown in Fig. [1]. Firstly, all old available free-air gravity anomalies at 800 points, where the sources of these data their number and distribution are well documented in many previous works as shown in (Amin et al., 2002, 2003; Hassouna, 2003) free-air gravity anomaly values at 267 points were obtained from BGI [Bureau Gravimetric International], where their observational mean stander deviation is [0.24mgal], while the standard deviation estimated for older gravity anomaly data distributed all over the whole territory of Egypt is [0.73mgal] on average, secondly Marine free-air gravity

anomalies at 31934 points. As can be seen from figure [1], free air gravity data distribution is not homogeneous over the land, with significant gaps, particularly in the eastern and western deserts, while it's approximately homogeneous distributed over the seas. In addition to 71 local gravity anomalies which are used in external accuracy check of the tailored EGXGM2016 as shown in figure [2].

TABLE (1)
The available used data

Item	Data No. after filtration
Gravity anomalies [old]	800
Gravity anomalies [BGI]	267
Marine gravity anomalies	31,934

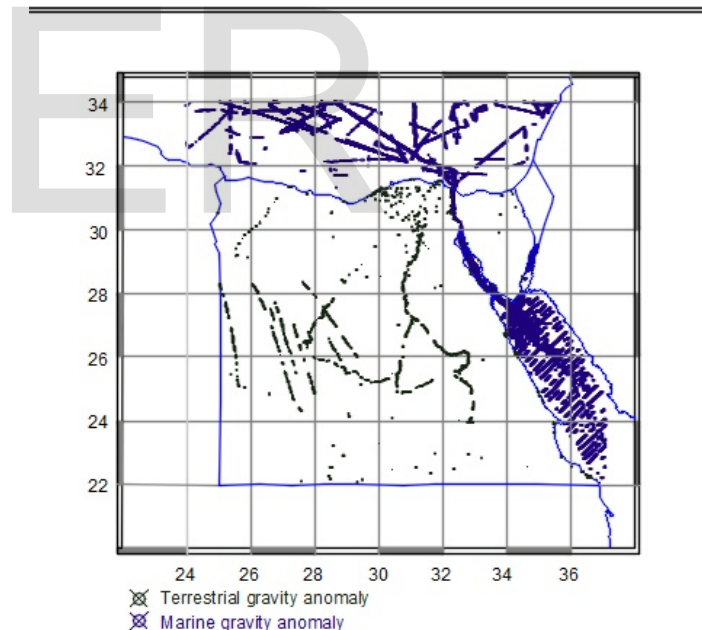


Fig. 1. Free air gravity anomaly [g_{fa}] distribution over

EGYPT

4. THE EXPERIMENTAL GRAVITY FIELD MODEL XGM2016

In December 2015, the United States National Geospatial-Intelligence Agency [NGA] has agreed to provide the Technical University of Munich [TUM] with a new, global 15'x15' grid of 'terrestrial' gravity anomaly area means. This grid incorporates the majority of NGA's new altimetric

and terrestrial survey data, as well benefiting from new procedures for processing this data. At this early stage, TUM has agreed to provide NGA with an independent assessment of this new data grid, in terms of its suitability for supporting an improved EGM. One outcome of this effort is the Experimental Gravity Field Model 2016 [XGM2016].

XGM2016 extend to spherical harmonic degree of 719, which is the maximum resolution supported by its 15'x15' terrestrial grid

For XGM2016, a significant focus will be the optimal combination of the new terrestrial data with the latest satellite gravity information. This includes 11 years of GRACE (2002-2013), and the entire GOCE mission (2009-2013). The combination is based on a full normal equation system up to the maximum degree (n=719) of the expansion. (Paillet al., 2017)

Table [2]: The parameter of EGM [XGM2016]

product_type	gravity_field
Modelname	XGM2016
earth_gravity_constant	3.9860044150e+14
radius	6.3781363000e+06
max_degree	719
errors	formal
norm	fully_normalized
tide_system	zero_tide

5. RESULTS

Table [3] shows the statistics of the residuals of discrete gravity anomaly data points, using the XGM2016 and EGXGM2016 with terrestrial free air gravity anomalies data points. It is clear how much the local information has been incorporated into the model tailored by collocation.

Table (4) shows the statistics of the residuals of discrete gravity anomaly data points, but regarding to the external accuracy by using [71 free air gravity anomaly] with the tailored model, namely, the EGXGM2016 model. Of course, much local features have been also introduced to this tailored model. Regarding both tables, the refinement is implied by the great smoothness of the residuals, in terms of the mean and standard deviation. In brief, the EGXGM2016 tailored models possess superior long to medium wavelength behaviors over the XGMT2016 model, regarding the spectral amount removed from the discrete gravity anomaly data, by the tailored model, compared to

the XGM2016 harmonic model. Obviously the EGXGM2016 model has improved behavior, over the original model.

Table (3) Statistics of the residual anomaly data sets from the discrete gravity anomaly data points (unit: mgal)

	Free air gravity anomaly	Min mgal	Max mgal	Mean mgal	Std. Dev. mgal
	The local Free Air (None)	-164.300	129.428	-37.610	41.117
$\delta \Delta g$	XGM2016	-98.400	123.010	-3.821	16.831
	EGXGM2016	-61.480	82.680	-1.560	10.500

Table (4) Statistics of the residual anomaly data sets from the discrete gravity anomaly data points for external accuracy check (unit: mgal)

	Free air gravity anomaly	Min mgal	Max mgal	Mean mgal	Std. Dev. mgal
	The local Free Air (None)	-49.970	72.770	-2.697	23.233
$\delta \Delta g$	XGM2016	-30.336	43.170	-0.577	15.186
	EGXGM2016	-54.277	57.811	-2.929	20.253

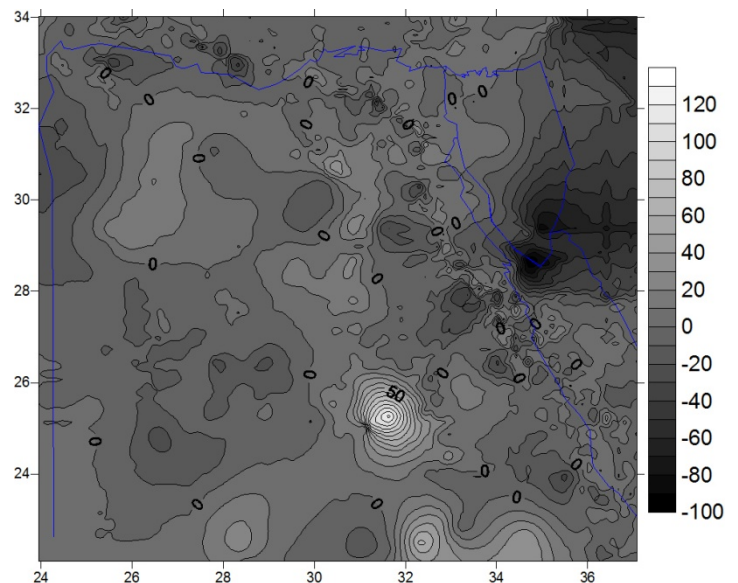


Fig.3 Δg_{fa} residuals referred to EGM XGM2016 with terrestrial data for the whole area of Egypt.

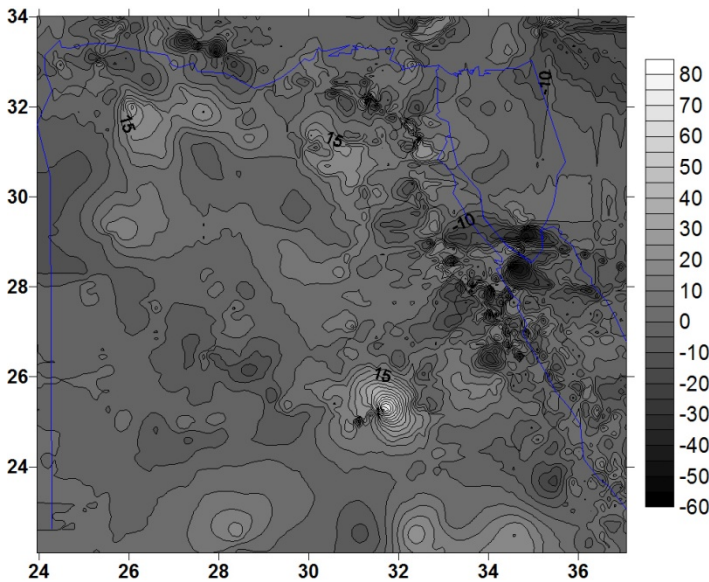


Fig.3 Δgfa residuals referred to Tailored EGXGM2016 with terrestrial data for the whole area of Egypt.

6. CONCLUSION

The results illustrated that the tailored model of XGM2016 denoted as EGXGM2016 has acceptable performance W.R.T the poor, lake and bad distribution of the local data, where its mean value is [1.56 mgal], the standard deviation is [10.50 mgal] and the range of the reduced gravity anomalies to EGXGM2016 compared with XGM2016 have lesser values by about [43.52 mgal] and the standard deviation is better by 62%. While the external accuracy by using [71 free air gravity anomaly data points] denoted that 54% lesser in terms of standard deviation.

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